Problem Sheet 2

The following problems involve equations and inequalities.

1. Find the smallest integer solution to the following equation:

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$$

2. Given that $u > 0$ and $v > 0$ what is the smallest possible value of $1/u + 1/v$ given that $u + v = 5$?

3. Find the range of values of x for which

$$\sqrt{x} + \frac{1}{\sqrt{x}} < 4\,,$$

   where $\sqrt{x}$ is the positive root.

4. If $0 < x < 1$ and $0 < y < 1$ then prove

$$x + y < 1 + xy$$

   .

5. Is the mean of the squares of two numbers greater than, or less than, the square of their means?

6. Find the maximum of

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$$

   where p, q and r are positive integers and

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1.$$

   Prove that it is indeed a maximum.

7. Find all positive integers x, y and z such that:

$$x + \cfrac{1}{y + \cfrac{1}{z}} = \frac{10}{7}$$